APPENDIX A

1 initialization

1.1 set c=0, j=0, and i=0

2 if ($0 \leq i \leq m$) e.g., within the range of S, then

2.1 if (group j not fully matched AND $i \leq m$), then

2.1.1 match event i against group j

2.1.2 increase i by 1

2.1.3 if all event categories in group j are matched BUT ω_j violated), then

2.1.3.1 make h to be the smallest index such that $T(e_h) \geq T(e_i) - \omega_j$

2.1.3.2 set i to h

2.1.3.3 remove all matches in group j

2.1.4 go to 2.1

2.2 if group j cannot be matched and $i > m$, then go to 3

2.3 else if β_{j-1} is violated, then

2.3.1 make i the smallest index such that $T(e_h) \geq$ (latest time in group j)- β_j

2.3.2 decrease j by 1 to rework the previous group

2.4 else // group j succeed

2.4.1 increase i so that $T(e_i) > \alpha_j$ + e.g., make i the latest time in group j

2.4.2 if $c > 0$, then

2.4.2.1 increase i so that $T(e_i) >$(earliest time in group j) in the last matched occurrence)

2.4.3 increase j by 1

2.5 if j equals g, i.e., current occurrence is fully matched, then

2.5.1 increase c by 1

2.5.2 remove event instances in current matched occurrence

2.5.3 reset j = 0

2.5.4 direct i to point to the event right after earliest event in the current matched occurrence

2.6 go to 2

3 report c

APPENDIX B

1 Method initialization

- 1.1 Determine $\varphi_{1,z}$ and λ_1
- 1.2 Determine δ
- 1.3 Set $\psi_e = 0$, $\psi_o = \psi_1$
- 1.4 Determine γ_1
- 1.5 Assume $\sigma = 1$
- 1.6 Assume $\mu^{-} = \delta + \psi_e$ and $\mu+ = \delta + \psi_o + \gamma_1$

2 if $\mu^{+} - \mu^{-} \leq \rho\mu^{-}$, then break, if not then

- 2.1 Determine $\varphi_{k,z}$ and λ_k for $k = 2\sigma, 2\sigma + 1$
- 2.2 If $2\sigma \geq x$, then go to 3
- 2.3 Update ψ_e
- 2.4 Update μ^{-}
- 2.5 If $\mu^{+} - \mu^{-} \leqq \rho\mu^{-}$, then go to 3
- 2.6 If $2\sigma + 1 \geq x$, then go to 3
- 2.7 Update ψ_o
- 2.8 Update μ^{+}
- 2.9 Set $\sigma = \sigma + 1$
- 2.10 Go to 2

3 Output μ^{+} and μ^{-}

APPENDIX C

1 Method initialization

1.1 Determine $\varphi_{1,z}$ and λ_1:

$$\varphi_{1,0} = 1$$

$$\varphi_{1,1} = -1$$

$$\lambda_1 = q^{2(y-x+1)}$$

1.2 Determine δ: $\delta = x - (q^{y-x+1}(1-q^{x}))/(1-q)$

1.3 Set $\psi_e = 0$, $\psi_o = \psi_1$: where

$$\psi_1 = q^{2y-2x+3}(1-q)$$

1.4 Determine γ_1

$$\gamma_1 = \lambda_1\left((q^2-q^x)/(1-q)-(q^4-q^{2x})(1-q^2)\right)$$

1.5 Set $\sigma = 1$

1.6 Set $\mu^- = \delta + \psi_e$ and $\mu+ = \delta + \psi_o + \gamma_1$

2 while ($\mu^+ - \mu^- \leq p\mu^-$), then break, if not then

2.1 Determine $\varphi_{k,z}$ and λ_k for $k = 2\sigma, 2\sigma + 1$:

$$\varphi_{k,0} = \sum_{z=1}^{k} -\varphi_{k,z}q^{z}$$

$$\varphi_{k,z} = -(\varphi_{k-1,z-1})/(1-q^{z})$$

$$\lambda_k = -\lambda_{k-1}(1-q^{k})q^{y-x+1}$$

2.2 If ($2\sigma \geq x$), then go to 3

2.3 Update ψ_e:

$$\text{increase } \psi_e \text{ by } \sum_{z=1}^{2\sigma-2}\left[\lambda_z \sum_{w=0}^{z} \varphi_{z,w}(q^{(2\sigma+1)(w+1)}+q^{2\sigma(w+1)})\right]$$

2.4 Update μ^-: Set $\mu^- = \delta + \psi_{2\sigma} + \gamma_{2\sigma}$

2.5 If ($\mu^+ - \mu^- \leq \rho\mu^-$), then go to 3

2.6 If ($2\sigma + 1 \geq x$), then go to 3

2.7 Update ψ_o:

$$\text{increase } \psi_0 \text{ by } \sum_{z=1}^{2\sigma-1}\left[\lambda_z \sum_{w=0}^{z} \varphi_{z,w}(q^{(2\sigma+1)(w+1)}+q^{2\sigma(w+1)})\right]$$

2.8 Update μ^+: Set $\mu^+ = \delta + \psi_{2\sigma+1} + \gamma_{2\sigma+1}$

2.8 Set $\sigma = \sigma + 1$

3 Output μ^+ and μ^-